# Nonanticommutative superspace
# and N= 1/2 WZ model

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## ABSTRACT

In these proceedings we review the main results concerning superspace geometries with nonanticommutative spinorial variables and field theories formulated on them. In particular, we report on the quantum properties of the WZ model formulated in the $N = 1/2$ nonanticommutative superspace.

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# 1 Introduction

Since the famous paper by Seiberg and Witten [1] it has been known that the low–energy dynamics of D3–branes in flat space, in the presence of a constant Neveu–Schwarz magnetic background is described by a $N = 4$ supersymmetric Yang–Mills theory defined on a noncommutative 4d spacetime, $[x^\mu, x^\nu] = i\Theta^{\mu\nu}$. Previous indications about the possibility to have noncommutative geometry from string theory or M(atrix) theory can be found in [2].

Supersymmetry plays a fundamental role in string theory. In particular, in the Green–Schwarz [3] or Berkovits [4] formulations of superstring the target space supersymmetry is made manifest and the superstring sigma-model action describes the dynamics of the coordinate variables of a ten dimensional superspace. It is then natural to investigate the possibility of embedding noncommutativity in superspace and defining superspace geometries where the non(anti)commutativity is extended to the spinorial coordinates.

Independently of a string context, a number of papers [5, 6, 7] have studied the possibility to define such supergeometries. In [7] a systematic construction of the most general nonanticommutative (NAC) superspace has been done by studying the compatibility between noncommutativity and supersymmetry. Among the results of that paper, it is important to mention the following: It has been proven that in Euclidean $N = (1, 1)$ four dimensional superspace a NAC generalization which is associative and compatible with supersymmetry can be given where nontrivial anticommutators $\{\theta_i^\alpha, \theta_j^\beta\}$ are turned on. Since the $N = 1$ euclidean superspace (rigorously called $N = (\frac{1}{2}, \frac{1}{2})$) superspace) can be defined by a suitable truncation of $N = (1, 1)$, one obtains the NAC generalization of euclidean $N = (\frac{1}{2}, \frac{1}{2})$ superspace with $\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta}$ constant. As proven in [7], the possibility to turn on nontrivial anticommutation relations for the spinorial coordinates by keeping the algebra associative is peculiar of the *euclidean* superspace. Associativity in Minkowski necessarily requires the spinors to be ordinary grassmannian variables.

In a string theory context, the main question is whether particular string configurations exist which, in the low energy limit, give rise to supersymmetric field theories defined on such superspace geometries. This question has been recently answered in a number of remarkable papers [8, 9, 10] where it has been shown that in the low energy limit a type IIB string with D3–branes compactified on a Calabi–Yau three–fold, in the presence of a self-dual, constant graviphoton background $F_{\alpha\beta}$ gives rise to the NAC superspace geometry $\{\theta^\alpha, \theta^\beta\} = \alpha'^2 F_{\alpha\beta}$. We note that the graviphoton is taken to be self–dual in order to avoid any back–reaction on the metric which remains flat. On the other hand, the graviphoton can be taken self–dual only in *euclidean* signature, consistently with the fact that only euclidean superspace allows for $\{\theta^\alpha, \theta^\beta\} \neq 0$. Similar results in ten dimensions have been obtained in [11].

In [9] the investigation of field theories defined on such geometries has been initiated. This requires the definition of a suitable graded Poisson structure associated to

the nontrivial algebra $\{\theta^\alpha, \theta^\beta\}$ and a corresponding star product in the space of smooth functions defined on superspace. As we will describe later, there may be different NAC generalizations of the $N = (\frac{1}{2}, \frac{1}{2})$ superspace according to which representation we choose for the supersymmetry charges and for the corresponding spinorial covariant derivatives. Moreover, there is more than one star product compatible with $\{\theta^\alpha, \theta^\beta\} \neq 0$.

The important observation is that in some NAC generalizations the algebra of superfields does not respect the complete $N = 1$ supersymmetry. From [9] and subsequent discussions [12, 13] it appears that a NAC but susy preserving generalization can be done only at the expense of a consistent definition of chirality. On the other hand, if we insist on keeping the ordinary definition of chirality and require the star product of two chirals to be chiral, then we are forced to dress the algebra of superfields with a star product which breaks half of the supersymmetry [9]. We are then led to consider field theories defined on $N = (\frac{1}{2}, 0)$ superspace (briefly called $N = 1/2$ superspace).

The simplest model one can study on such NAC geometries is the $N = 1/2$ generalization of the WZ model describing the dynamics of a scalar superfield self-interacting with a cubic superpotential $\Phi * \Phi * \Phi$. At the classical level it turns out to be the ordinary WZ model perturbed by a soft-breaking term $C^2 \int d^4x F^3(x)$, where $C^2$ is the square of the anticommutation parameter and $F$ is the highest component of the chiral superfield. At the quantum level, renormalizability properties of the model have been studied in [14, 15, 16]. The model as it stands is not renormalizable since at one–loop a divergent contribution arises proportional to $C^2 \int d^4x F^2(x)$. However, in [16] it has been proven that the addition of this extra term in the classical lagrangian makes the theory renormalizable up to two loops. Further investigations have been carried on and a proof of the renormalizability of the model with the extra $F^2$ term at any order of perturbation theory has been eventually given in [17].

We note that the renormalizability of the NAC WZ model is not obvious from power counting due to the appearance of a dimensionful constant (the anticommutation parameter $C^{\alpha\beta}$) and, consequently, of irrelevant deformations with scaling dimensions greater than four. However, in [18] a nice argument has been given to prove the renormalizability of the model by power counting. It is based on the observation that in euclidean space, due to the lack of h.c. relations between the spinorial variables $\theta^\alpha$ and $\bar{\theta}^{\dot\alpha}$ one can make a reassignement of the scale dimensions of the spinorial coordinates in such a way to have $C^{\alpha\beta}$ dimensionless and the associated $F^3$ deformation as a marginal operator.

In the next Section we review the construction of the $N = 1/2$ superspace and discuss the issue of half–breaking of supersymmetry. In Section 3 we report the results of [16] about the two–loop renormalization of the NAC WZ model and discuss as an extra $F^2$ deformation is needed to guarantee the renormalizability of the model. Finally, in the conclusions we mention further developments and possible future lines of investigation.

## 2 The $N = 1/2$ nonanticommutative superspace

Following [7] one can find the most general algebra for the coordinates of a flat superspace compatible with supersymmetry by imposing the covariance of the fundamental algebra under translations and supertranslations. If we work in Minkowski signature, the extra condition for the algebra of the coordinates to be associative brings in quite severe constraints which allow, as the only nontrivial commutators, $[x, \theta]$, $[x, \bar{\theta}]$ and $[x, x]$. However, it was shown in [7] that euclidean signature is less restrictive and a NAC superspace with $\{\theta, \theta\}$ different from zero can be defined consistently with associativity.

Rigorously, a superspace with euclidean signature can be defined only when extended susy is present because of the impossibility of assigning consistent reality conditions for the pair of Weyl fermions $\theta_\alpha$, $\bar{\theta}_{\dot{\alpha}}$ (for a detailed review on the subject see for instance [19]). However, in the $N = 1$ case one can still define a superspace with euclidean signature by temporarily doubling the fermionic degrees of freedom and choosing nonstandard conjugation rules among them (for a detailed discussion, see [13]).

We review the results of [7] on $N = (1, 1)$ euclidean superspace in view of the fact that eventually we will make a truncation to $N = (\frac{1}{2}, \frac{1}{2})$. We describe the $N = (1, 1)$ superspace by coordinates $(x^{\alpha\dot{\alpha}}, \theta^\alpha, \bar{\theta}^\alpha, \theta^{\dot{\alpha}}, \bar{\theta}^{\dot{\alpha}})$ subject to the complex conjugation conditions $(\theta^\alpha)^* = \theta^\beta \epsilon_{\beta\alpha}$, $(\bar{\theta}^\alpha)^* = \bar{\theta}^\beta \epsilon_{\beta\alpha}$ and the same for dot variables. These are not the standard conjugation rules for spinorial coordinates since the $*$–operation squares to $-1$. However, these are the conjugation rules compatible with truncation to $N = (\frac{1}{2}, \frac{1}{2})$. The important point is that in euclidean signature there are no h.c. relations between $\theta^\alpha$ and $\theta^{\dot{\alpha}}$.

The structure of the NAC algebra depends on the representation we choose for susy charges and covariant derivatives. In [7] we chose a nonchiral representation which brought to a NAC geometry where the algebra of covariant derivatives and supercharges were both deformed. In [9] an alternative proposal was made which uses the chiral representation for derivatives and charges (we consider only the left sector and use the conventions of [20])

$$
\begin{aligned}
Q_{\dot{\alpha}} &= i(\partial_{\dot{\alpha}} - i\theta^\alpha \partial_{\alpha\dot{\alpha}}) \quad, \quad Q_\alpha = i\partial_\alpha \\
D_{\dot{\alpha}} &= \partial_{\dot{\alpha}} \qquad\qquad\quad, \quad D_\alpha = \partial_\alpha + i\theta^{\dot{\alpha}} \partial_{\alpha\dot{\alpha}}
\end{aligned}
\tag{2.1}
$$

In principle the NAC algebra consistent with susy and associativity is of the form

$$
\begin{aligned}
\{\theta^\alpha, \theta^\beta\} &= 2C^{\alpha\beta} \qquad \{\theta^{\dot{\alpha}}, \theta^\beta\} = \{\theta^{\dot{\alpha}}, \theta^{\dot{\beta}}\} = 0 \\
[x^{\alpha\dot{\alpha}}, \theta^\beta] &= -2iC^{\alpha\beta}\theta^{\dot{\alpha}} \qquad [x^{\alpha\dot{\alpha}}, x^{\beta\dot{\beta}}] = 2\theta^{\dot{\alpha}} C^{\alpha\beta}\theta^{\dot{\beta}}
\end{aligned}
\tag{2.2}
$$

but a suitable change of variable $y^{\alpha\dot{\alpha}} = x^{\alpha\dot{\alpha}} - i\theta^\alpha\theta^{\dot{\alpha}}$ avoids dealing with noncommuting $x$'s. Therefore the superspace coordinates $(y^{\alpha\dot{\alpha}}, \theta^\alpha, \bar{\theta}^\alpha, \theta^{\dot{\alpha}}, \bar{\theta}^{\dot{\alpha}})$ satisfy

$$
\{\theta^\alpha, \theta^\beta\} = C^{\alpha\beta} \qquad\qquad \text{the rest} = 0
\tag{2.3}
$$

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In this case the algebra of the covariant spinor derivatives is not modified, while

$$\{Q_\alpha, Q_\beta\}_* = 0 \quad , \quad \{Q_\alpha, Q_{\dot\alpha}\}_* = i\partial_{\alpha\dot\alpha} \quad , \quad \{Q_{\dot\alpha}, Q_{\dot\beta}\}_* = 2C^{\alpha\beta}\partial_{\alpha\dot\alpha}\partial_{\beta\dot\beta} \qquad (2.4)$$

Therefore the supersymmetry is explicitly broken to $N = (\frac{1}{2}, 0)$ [9] on the class of smooth functions defined on this superspace. We note that the susy-breaking term is quadratic in the bosonic derivatives, so it does not spoil the previous statement about consistency of (2.3) with supersymmetry invariance of the fundamental algebra of the coordinates.

Following Seiberg we realize the NAC geometry on the smooth superfunctions by introducing the nonanticommutative (but associative) product

$$\phi * \psi = \phi e^{-\overleftarrow{\partial}_\alpha C^{\alpha\beta}\overrightarrow{\partial}_\beta}\psi = \phi\psi - \phi\overleftarrow{\partial}_\alpha C^{\alpha\beta}\overrightarrow{\partial}_\beta\psi - \frac{1}{2}C^2\partial^2\phi\partial^2\psi \qquad (2.5)$$

where we have defined $C^2 = C^{\alpha\beta}C_{\alpha\beta}$. Since the covariant derivatives (2.1) are still derivations for this product, if we define (anti)chiral superfields as usual the classes of (anti)chirals are still closed. However, this product explicitly breaks the $\bar{Q}$–supersymmetry, being defined in terms of noncovariant spinor derivatives.

Before closing this section we note that, if we were to use the antichiral representation for charges and covariant derivatives (basically by interchanging the definitions of $Q$'s and $D$'s in (2.1)) we would still obtain a NAC generalization of superspace described by the algebra (2.3) but in this case the algebra of derivatives would get deformed as in (2.4), while the susy charges would satisfy the ordinary anticommutation rules. Moreover, since in this representation $D_\alpha = \partial_\alpha$, the $*$–product would be naturally defined in terms of covariant derivatives, so avoiding explicit breaking of supersymmetry. One might conclude that in this representation supersymmetry is not broken at all. However, the modification of the anticommutation relations between covariant derivatives makes it difficult to proceed and consistently define (anti)chiral representations. This issue certainly requires more investigation.

# 3 The $N = 1/2$ WZ model

Given the $N = (1/2, 0)$ *euclidean* superspace and the algebra of the smooth functions defined on it endowed with the $*$–product (2.5), we may define NAC supersymmetric field theories by promoting ordinary lagrangians to NAC ones where the ordinary products have been substituted with $*$–products. The simplest model we can consider is the NAC generalization of the WZ model described by the action

$$S = \int d^8z\bar{\Phi}\Phi - \frac{m}{2}\int d^6z\Phi^2 - \frac{\bar{m}}{2}\int d^6\bar{z}\bar{\Phi}^2$$
$$- \frac{g}{3}\int d^6z\Phi * \Phi * \Phi - \frac{\bar{g}}{3}\int d^6\bar{z}\bar{\Phi} * \bar{\Phi} * \bar{\Phi} \qquad (3.1)$$

This action is generically complex since no h.c. relations are assumed for fields, masses and couplings.

Performing the expansion of the star product as in (2.5) and neglecting total superspace derivatives, the cubic interaction terms reduce to the usual WZ interactions augmented by the nonsupersymmetric component term $\frac{g}{6}C^2 \int d^4 x F^3$ [9], where $F = D^2 \Phi|$. The action can be written as

$$
S = \int d^8 z \, \bar{\Phi} \Phi - \frac{m}{2} \int d^6 z \, \Phi^2 - \frac{\bar{m}}{2} \int d^6 \bar{z} \, \bar{\Phi}^2 - \frac{g}{3} \int d^6 z \, \Phi^3 - \frac{\bar{g}}{3} \int d^6 \bar{z} \, \bar{\Phi}^3
$$
$$
+ \frac{g}{6} \int d^8 z \, U (D^2 \Phi)^3 \tag{3.2}
$$

where we have introduced the external, constant spurion superfield [21] $U = \theta^2 \bar{\theta}^2 C^2$ in order to deal with a well–defined superspace expression for the extra term proportional to the NAC parameter. This allows us to use standard supergraphs techniques to perform perturbative calculations.

The action in components reads ($\Phi| = \phi$, $D_\alpha \Phi| = \psi_\alpha$, $D^2 \Phi| = F$ and analogously for the antichiral components)

$$
S = \int d^4 x \Big[ \phi \Box \bar{\phi} + F \bar{F} - GF - \bar{G} \bar{F} + \frac{g}{6} C^2 F^3
$$
$$
+ \psi^\alpha i \partial_\alpha^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} - \frac{m}{2} \psi^\alpha \psi_\alpha - \frac{\bar{m}}{2} \bar{\psi}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} - g \phi \psi^\alpha \psi_\alpha - \bar{g} \bar{\phi} \bar{\psi}^{\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} \Big] \tag{3.3}
$$

where we have defined

$$
G = m\phi + g\phi^2 \qquad \bar{G} = \bar{m}\bar{\phi} + \bar{g}\bar{\phi}^2 \tag{3.4}
$$

The auxiliary fields $F$ and $\bar{F}$ satisfy the algebraic equations of motion (EOM)

$$
F = \bar{G} \qquad , \qquad \bar{F} = G - \frac{g}{2} C^2 F^2 = G - \frac{g}{2} C^2 \bar{G}^2 \tag{3.5}
$$

In [16] the perturbative evaluation of the effective action up to two loops has been performed. Here we report the basic results and refer the reader to that reference for details of the calculation.

The procedure we have applied is the following: We have performed quantum–background splitting by setting $\Phi \to \Phi + \Phi_q$ and integrating out the quantum fluctuations $\Phi_q$. From the expansion of the action (3.2) we have read the Feynman rules for vertices and propagators. It is important to note that the propagators are the ordinary ones since the quadratic part of the action is not modified by the $*$–product. Instead, two new quadratic and cubic vertices appear from the $U$ term. At a given loop order we have drawn all supergraph configurations with the corresponding chiral and antichiral derivatives from the vertices and the propagators. Then we have performed $D$–algebra to reduce the supergraphs to

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ordinary momentum diagrams. We have worked in dimensional regularization and minimal subtraction scheme. We have used BPHZ renormalization techniques which amounts to start with the classical action written in terms of renormalized quantities and order by order perform the subtraction of subdivergences directly on the diagrams. Finally, in the counterterms we need add to the action to remove divergent contributions, we have made repeated use of the EOM (3.5) for the auxiliary field $F$. At any loop order this is justified by the important observation that, due to the particular form of the propagators, the insertion of counterterms proportional to $\bar{G} = \bar{m}\bar{\phi} + \bar{g}\bar{\phi}^2$ that one performs into higher loop diagrams in order to cancel subdivergences is *equivalent* to the insertion of counterterms proportional to $F$ (see [16] for details).

Applying this strategy, the main results we have obtained are the following: At one loop we have the ordinary self–energy $\bar{\Phi}\Phi$ divergent diagram which induces a wave function renormalization, plus two new divergent contributions proportional to $F^2$ and $F^3$ coming from diagrams with one $U$ insertion and one external $\Phi$, and external $\Phi$, $\bar{\Phi}$ and $\Phi$, $\bar{\Phi}$, $\bar{\Phi}$, respectively. The $F^3$ divergence induces a renormalization of the spurion $U$, whereas the $F^2$ does not have a classical counterpart and makes the model (3.1) not renormalizable. Thus we have considered a modified action with the addition of $F^2$ and $F$ terms (once we have $F^2$ it is obvious that we will generate tadpoles) which in superspace language reads

$$S_r = S + k_1 \bar{m}^4 \int d^8 z\, U D^2 \Phi + k_2 \bar{m}^2 \int d^8 z\, U (D^2 \Phi)^2 \tag{3.6}$$

with $S$ given in (3.2). Starting from this action, we have computed the divergent contributions up to two loops and found the following results:

- Divergent diagrams contain at most one $U$ vertex

- Divergences are always logarithmic

- The antiholomorphic part of the action does not get renormalized. Moreover, the ordinary part of the action (the one independent of $C^{\alpha\beta}$) does not receive contributions proportional to the anticommutation parameter

- In components the general structures of the divergent terms are

$$C^2 \int d^4 x \left[ a_0 F + a_1 F\bar{G} + a_2 F^2 \bar{G} + a_3 F\bar{G}^2 + a_4 F + a_5 F^2 + a_6 F^3 \right] \tag{3.7}$$

where $\bar{G}$ is given in (3.4).

Now, using the classical equation of motion, $F = \bar{G}$, all the divergent terms assume the form $F, F^2, F^3$. In conclusion, we have proved that the counterterms $F, F^2, F^3$ (or their superspace expressions in terms of $U$ superfield) are sufficient to renormalize the theory (3.6) up to two loops.

In [16] the two–loop beta functions for the couplings of the theory have been also computed. Even if we expect them to be affected by scheme dependence it is interesting to note that nontrivial fixed points for the NAC parameter might exist.

# 4   Conclusions

In these proceedings we have reviewed the results of [16] concerning the two–loop renormalizability of the NAC WZ model described by the action (3.6). The main result is the appearance of the extra $F$ and $F^2$ terms in the classical action which, as shown in [16], are sufficient to make the theory renormalizable at two loops. Further investigations have been carried on in [17] where it has been proven that the addition of these extra terms is sufficient to make the theory renormalizable at any loop order. A basic ingredient of the proof is the existence of two global U(1) (pseudo)symmetries which constrain the structure of the counterterms.

We note that the $F^2$ term cannot be written as the $*$–product of anything. Therefore, one might worry about the presence of this extra term as deforming the definition of $*$–product at quantum level. A possible interpretation of this term has been discussed in [22]. In a string theory context it would be nice to understand the origin of these extra contributions.

Our approach could be suitable for performing perturbative calculations in NAC Yang–Mills theories (indications about renormalizability of those theories are contained in [23, 18]). In that case, an explicit expression for the NAC action has been worked out in components in the WZ gauge and one–loop calculations have been done [24]. In order to implement ordinary superspace techniques and push the calculations beyond one loop it should be necessary to find an expression for the NAC action in the gauge of superspace rather than in the WZ gauge.

Other interesting generalizations of our analysis would concern susy field theories defined in superspaces where also the bosonic coordinates would be noncommuting.

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